

HOW COMPLICATED CAN THE CIRCUIT NETWORK BE WHEN USING CircuitNAV?

Boundaries of Symbolic Circuit Analysis



CircuitNAV's Capability

- CircuitNAV is a symbolic circuit analysis tool
- It generates systems of equations for node voltages based on Kirchhoff's Current Law
- CircuitNAV uses SymPy, a computer algebra system, to solve the systems of equations
- Theoretically, the tool does not have a limitation on how many nodes the circuit netlist can have
- Practically, CircuitNAV does have a boundary - how come?

Limitations on Algebraic Solutions for Systems of Equations

- In theory there is no limit on the size of a system or the number of unknowns and equations
- SymPy does not have a limit on the size either
- To get the simplified solution, SymPy could take a significant amount of time when the size grows larger.

system size # of unknowns # of equations	# of product terms	SymPy Calculation Time(sec)
2	2 (2!)	1
3	6 (3!)	1
4	24 (4!)	7
5	120 (5!)	120 (2 min)
6	720 (6!)	3000 (50 min)

Visual Sense of the Solutions for System Size of Two and Three

For two equations and two unknowns, the solution has a sum of two product terms both in numerator and denominator.

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Two Unknowns, [x0, x1]; Two Equations:
a00*x0 + a01*x1 = b0
a10*x0 + a11*x1 = b1

SymPy Solution (takes an i7 CPU less than 1 sec):
x0 = (-a01*b1 + a11*b0)/(a00*a11 - a01*a10)
x1 = (a00*b1 - a10*b0)/(a00*a11 - a01*a10)

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Three Unknowns, [x0, x1, x2]; Three Equations:
a00*x0 + a01*x1 + a02*x2 = b0
a10*x0 + a11*x1 + a12*x2 = b1
a20*x0 + a21*x1 + a22*x2 = b2

SymPy Solution (takes an i7 CPU about 1 sec):
x0 = (b0*(a11*a22 - a12*a21) - b1*(a01*a22 - a02*a21) + b2*(a01*a12 - a02*a11))/(a00*a11*a22 - a00*a12*a21 - a01*a10*a22 + a01*a12*a20 + a02*a10*a21 - a02*a11*a20)
x1 = (-b0*(a10*a22 - a12*a20) + b1*(a00*a22 - a02*a20) - b2*(a00*a12 - a02*a10))/(a00*a11*a22 - a00*a12*a21 - a01*a10*a22 + a01*a12*a20 + a02*a10*a21 - a02*a11*a20)
x2 = (b0*(a10*a21 - a11*a20) - b1*(a00*a21 - a01*a20) + b2*(a00*a11 - a01*a10))/(a00*a11*a22 - a00*a12*a21 - a01*a10*a22 + a01*a12*a20 + a02*a10*a21 - a02*a11*a20)
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For three equations and three unknowns, the solution has a sum of six product terms both in numerator and denominator.

Visual Sense of the Solutions for System Size of Four

For four equations and four unknowns, the solution has a sum of 24 product terms both in the numerator and denominator.

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Four Unknowns, [x0, x1, x2, x3]; Four Equations:

$$a00*x0 + a01*x1 + a02*x2 + a03*x3 = b0$$

$$a10*x0 + a11*x1 + a12*x2 + a13*x3 = b1$$

$$a20*x0 + a21*x1 + a22*x2 + a23*x3 = b2$$

$$a30*x0 + a31*x1 + a32*x2 + a33*x3 = b3$$

SymPy Solution (takes an i7 CPU about 7 sec):

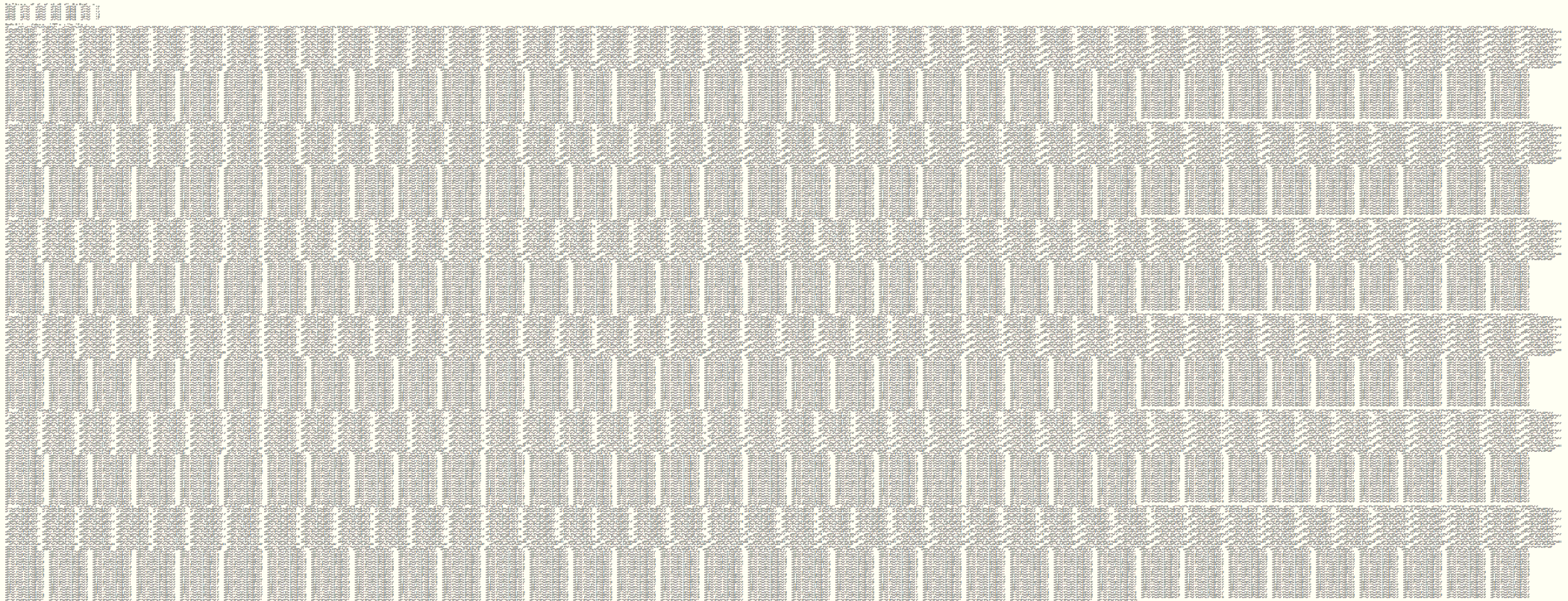
$$\begin{aligned} x0 &= (b0*(a11*a22*a33 - a11*a23*a32 - a12*a21*a33 + a12*a23*a31 + a13*a21*a32 - a13*a22*a31) - b1*(a01*a22*a33 - a01*a23*a32 - a02*a21*a33 + a02*a23*a31 + a03*a21*a32 - a03*a22*a31) + b2*(a01*a12*a33 - a01*a13*a32 - a02*a11*a33 + a02*a13*a31 + a03*a11*a32 - a03*a12*a31) - b3*(a01*a12*a23 - a01*a13*a22 - a02*a11*a23 + a02*a13*a21 + a03*a11*a22 - a03*a12*a21))/(a00*a11*a22*a33 - a00*a11*a23*a32 - a00*a12*a21*a33 + a00*a12*a23*a31 + a00*a13*a21*a32 - a00*a13*a22*a31 - a01*a10*a22*a33 + a01*a10*a23*a32 + a01*a12*a20*a33 - a01*a12*a23*a30 - a01*a13*a20*a32 + a01*a13*a22*a30 + a02*a10*a21*a33 - a02*a10*a23*a31 - a02*a11*a20*a33 + a02*a11*a23*a30 + a02*a13*a20*a31 - a02*a13*a21*a30 - a03*a10*a21*a32 + a03*a10*a22*a31 + a03*a11*a20*a32 - a03*a11*a22*a30 - a03*a12*a20*a31 + a03*a12*a21*a30) \\ x1 &= (-b0*(a10*a22*a33 - a10*a23*a32 - a12*a20*a33 + a12*a23*a30 + a13*a20*a32 - a13*a22*a30) + b1*(a00*a22*a33 - a00*a23*a32 - a02*a20*a33 + a02*a23*a30 + a03*a20*a32 - a03*a22*a30) - b2*(a00*a12*a33 - a00*a13*a32 - a02*a10*a33 + a02*a13*a30 + a03*a10*a32 - a03*a12*a30) + b3*(a00*a12*a23 - a00*a13*a22 - a02*a10*a23 + a02*a13*a20 + a03*a10*a22 - a03*a12*a20))/(a00*a11*a22*a33 - a00*a11*a23*a32 - a00*a12*a21*a33 + a00*a12*a23*a31 + a00*a13*a21*a32 - a00*a13*a22*a31 - a01*a10*a22*a33 + a01*a10*a23*a32 + a01*a12*a20*a33 - a01*a12*a23*a30 - a01*a13*a20*a32 + a01*a13*a22*a30 + a02*a10*a21*a33 - a02*a10*a23*a31 - a02*a11*a20*a33 + a02*a11*a23*a30 + a02*a13*a20*a31 - a02*a13*a21*a30 - a03*a10*a21*a32 + a03*a10*a22*a31 + a03*a11*a20*a32 - a03*a11*a22*a30 - a03*a12*a20*a31 + a03*a12*a21*a30) \\ x2 &= (b0*(a10*a21*a33 - a10*a23*a31 - a11*a20*a33 + a11*a23*a30 + a13*a20*a31 - a13*a21*a30) - b1*(a00*a21*a33 - a00*a23*a31 - a01*a20*a33 + a01*a23*a30 + a03*a20*a31 - a03*a21*a30) + b2*(a00*a11*a33 - a00*a13*a31 - a01*a10*a33 + a01*a13*a30 + a03*a10*a31 - a03*a11*a30) - b3*(a00*a11*a23 - a00*a13*a21 - a01*a10*a23 + a01*a13*a20 + a03*a10*a21 - a03*a11*a20))/(a00*a11*a22*a33 - a00*a11*a23*a32 - a00*a12*a21*a33 + a00*a12*a23*a31 + a00*a13*a21*a32 - a00*a13*a22*a31 - a01*a10*a22*a33 + a01*a10*a23*a32 + a01*a12*a20*a33 - a01*a12*a23*a30 - a01*a13*a20*a32 + a01*a13*a22*a30 + a02*a10*a21*a33 - a02*a10*a23*a31 - a02*a11*a20*a33 + a02*a11*a23*a30 + a02*a13*a20*a31 - a02*a13*a21*a30 - a03*a10*a21*a32 + a03*a10*a22*a31 + a03*a11*a20*a32 - a03*a11*a22*a30 - a03*a12*a20*a31 + a03*a12*a21*a30) \\ x3 &= (-b0*(a10*a21*a32 - a10*a22*a31 - a11*a20*a32 + a11*a22*a30 + a12*a20*a31 - a12*a21*a30) + b1*(a00*a21*a32 - a00*a22*a31 - a01*a20*a32 + a01*a22*a30 + a02*a20*a31 - a02*a21*a30) - b2*(a00*a11*a32 - a00*a12*a31 - a01*a10*a32 + a01*a12*a30 + a02*a10*a31 - a02*a11*a30) + b3*(a00*a11*a22 - a00*a12*a21 - a01*a10*a22 + a01*a12*a20 + a02*a10*a21 - a02*a11*a20))/(a00*a11*a22*a33 - a00*a11*a23*a32 - a00*a12*a21*a33 + a00*a12*a23*a31 + a00*a13*a21*a32 - a00*a13*a22*a31 - a01*a10*a22*a33 + a01*a10*a23*a32 + a01*a12*a20*a33 - a01*a12*a23*a30 - a01*a13*a20*a32 + a01*a13*a22*a30 + a02*a10*a21*a33 - a02*a10*a23*a31 - a02*a11*a20*a33 + a02*a11*a23*a30 + a02*a13*a20*a31 - a02*a13*a21*a30 - a03*a10*a21*a32 + a03*a10*a22*a31 + a03*a11*a20*a32 - a03*a11*a22*a30 - a03*a12*a20*a31 + a03*a12*a21*a30) \end{aligned}$$



# Visual Sense of the Solutions for System Size of Six

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For six equations and six unknowns, the solution has a sum of 720 product terms both in numerator and denominator.

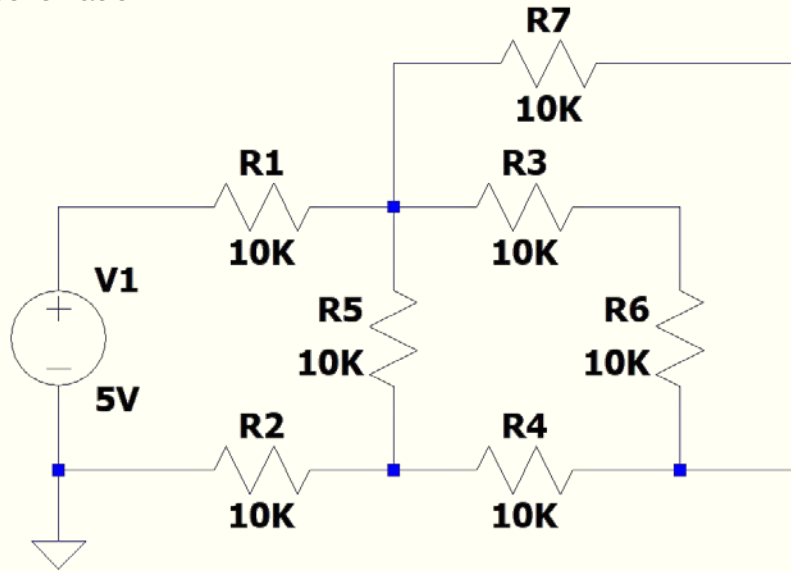


It takes an i7 CPU almost an hour to get the simplified solution!

# How Complicated can the Circuit be when using CircuitNAV?

- For a four node circuit, the algebraic solution could be messy.

- The schematic



- The solution:

The netlist:

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R1 N001 N003 10K
R2 N005 0 10K
R3 N004 N001 10K
R4 N002 N005 10K
R5 N005 N001 10K
R6 N002 N004 10K
V1 N003 0 5V
R7 N002 N001 10K
    
```

- When the netlist has five, six, or more nodes, the algebraic analysis may take minutes to hours to get the simplified result
- The result may not be algebraically meaningful, and hard for humans to make any sense out of it
- Numerical solutions might be the way to go



# The Conclusion

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- Zooming out on the solutions presented, we conclude:
  - When using the symbolic circuit analysis tool, e.g. CircuitNAV, be cautious when the circuit has more than five nodes. The solution might be too complicated to understand.
  - The limitation does not come from the tool, but from the sophisticated algebraic solution itself.